

Modern Physics Letters A
 © World Scientific Publishing Company

DIFFERENTIAL CROSS SECTION ANALYSIS IN KAON PHOTOPRODUCTION USING ASSOCIATED LEGENDRE POLYNOMIALS

P. T. P. HUTAURUK, D. G. IRELAND and G. ROSNER

*Department of Physics and Astronomy, University of Glasgow,
 Glasgow, G12 8QQ, Scotland, United Kingdom.
 p.hutauruk@physics.gla.ac.uk*

Received (Day Month Year)

Revised (Day Month Year)

Angular distributions of differential cross sections from the latest CLAS data sets⁶, for the reaction $\gamma + p \rightarrow K^+ + \Lambda$ have been analyzed using associated Legendre polynomials. This analysis is based upon theoretical calculations in Ref.¹ where all sixteen observables in kaon photoproduction can be classified into four Legendre classes. Each observable can be described by an expansion of associated Legendre polynomial functions. One of the questions to be addressed is how many associated Legendre polynomials are required to describe the data. In this preliminary analysis, we used data models with different numbers of associated Legendre polynomials. We then compared these models by calculating posterior probabilities of the models. We found that the CLAS data set needs no more than four associated Legendre polynomials to describe the differential cross section data. In addition, we also show the extracted coefficients of the best model.

Keywords: Kaon Photoproduction; Associated Legendre Polynomial.

PACS Nos.: include PACS Nos.

1. Introduction

Significant information on the structure of the nucleon can be obtained by studying its excitation spectrum. Over the last few decades, a large amount information about the spectrum of the nucleon has been collected. Most of this information has been extracted from pion-induced and pion photoproduction reactions. However, pionic reactions may have biased the information on the existence of certain resonances. Constituent quark model calculations predict a much richer resonance spectrum than has been observed in pion production experiments³. Predicted resonances which have not been observed are called "missing" resonances. Instead, the constituent quark model also predicts that these "missing" resonances may couple strongly to KA and $K\Sigma$ channels or other final states involving vector mesons^{3,4,5}. Since performing kaon-hyperon, kaon-nucleon or hyperon-nucleon scattering experiments is a daunting task, kaon photoproduction on the nucleon appears to be a

2 *P. T. P. HUTAURUK, et al.*

good alternative solution ^{4,5}.

Experiments on kaon photoproduction and electroproduction started in the 1960s. However the old experimental data are often inconsistent and have large error bars. In recent years a large amount of data for kaon photoproduction has been collected. High statistics data from CLAS, for differential cross sections, recoil polarization, C_x and C_z double polarizations for the reaction $\gamma + p \rightarrow K^+ + \Lambda$ have been published ^{6,7}. Additional experimental data have also been measured by SAPHIR ^{8,9,10}, LEPS ^{11,12} and GRAAL ¹³.

Several previous analyses have been applied to the results of these experiments, such as Isobar models ^{4,5,14,15,16} and Coupled channel models ^{17,18,19}. However different theoretical model calculations often produce very different predictions.

In Ref. ¹ all sixteen observables in kaon photoproduction were shown to be classified into the classes $\mathcal{L}_0(\hat{\mathbf{I}}; \hat{\mathbf{E}}; \hat{\mathbf{C}}_{\mathbf{z}'}; \hat{\mathbf{L}}_{\mathbf{z}'})$, $\mathcal{L}_{1a}(\hat{\mathbf{P}}; \hat{\mathbf{H}}; \hat{\mathbf{C}}_{\mathbf{x}'}; \hat{\mathbf{L}}_{\mathbf{x}'})$, $\mathcal{L}_{1b}(\hat{\mathbf{T}}; \hat{\mathbf{F}}; \hat{\mathbf{O}}_{\mathbf{x}'}; \hat{\mathbf{T}}_{\mathbf{z}'})$ and $\mathcal{L}_2(\hat{\mathbf{S}}; \hat{\mathbf{G}}; \hat{\mathbf{O}}_{\mathbf{z}'}; \hat{\mathbf{T}}_{\mathbf{x}'})$, where each class is an expansion in a different set of associated Legendre polynomials. What is not apparent is how many terms in each expansion are required. This work attempts to address the issue by examining data models with different numbers of terms, and calculating which one has the greatest posterior probability. In this article we only focus on the differential cross section observables, which are described by the associated Legendre class \mathcal{L}_0 .

2. Analysis Procedure

2.1. Data Model

We construct data models based on Legendre class \mathcal{L}_0 . These data models can be written compactly as follows:

$$M_L^{\mathcal{L}_0} = \sum_{l=0}^{l=L} A_l P_{l0}(\cos \theta). \quad (1)$$

where $M_L^{\mathcal{L}_0}$ is the data model, and A_l and $P_{l0}(\cos \theta)$ are the coefficients and associated Legendre polynomials. Each data model therefore has a different “order” or maximum number of polynomials. Our task is to find the most likely order.

2.2. Model Comparison

To determine the best model, we evaluate the posterior probability ²⁰ for each data model. The ratio of the probabilities for M_L and M_0 can be written, using Bayes theorem, as follows :

$$R = \frac{P(M_L|D)}{P(M_0|D)} = \frac{P(D|M_L)}{P(D|M_0)} \times \frac{P(M_L)}{P(M_0)}. \quad (2)$$

where $P(M_L|D)$ is the posterior for the M_L model, $P(D|M_L)$ is the probability that the data would be obtained, assuming M_L to be true (the likelihood). With

no prior prejudice as to which variant is correct, we obtain the ratio of likelihoods:

$$R = \frac{P(D|M_L)}{P(D|M_0)}$$

The likelihood $P(D|M_L)$ is an integral over the joint likelihood $P(D, \{A_l\}|M_L)$, where $\{A_l\}$ represents a set of free parameters:

$$\begin{aligned} P(D|M_L) &= \int \dots \int P(D, \{A_l\}|M_L) d^L A_l, \\ &= \int \dots \int P(D|\{A_l\}, M_L) P(\{A_l\}|M_L) d^L A_l. \end{aligned} \quad (3)$$

The function $P(A_l|M_L)$ is the prior probability that the parameters take on specific values. We assume that each parameter A_l lies in the range $A_l^{min} \leq A_l \leq A_l^{max}$, and we can write the prior as the reciprocal of the volume of a hypercube in parameter search space as $P(\{A_l\}|M_L) = \frac{1}{\prod^L (A_l^{max} - A_l^{min})}$. If the errors in the data points are Gaussian, it can be shown that $P(D|\{A_l\}, M_L) \propto \exp\left(-\frac{\chi^2}{2}\right)$, where χ^2 is the sum of squared residuals. Using a Taylor series expansion about the minimum χ^2 , $\chi^2 \approx \chi_{min}^2 + \frac{1}{2}(\mathbf{X} - \mathbf{X}_0)^T \nabla^2 \chi^2 (\mathbf{X} - \mathbf{X}_0) + \dots$, we can write an approximate form for the likelihood:

$$P(D|M_L) \propto \frac{L!(4\pi)^L}{\prod^L (A_l^{max} - A_l^{min}) \times \sqrt{\text{Det}(\nabla \nabla \chi^2)}} \exp\left(-\frac{\chi_{min}^2}{2}\right). \quad (4)$$

where L is the dimension of the integral and $(\text{Det}(\nabla \nabla \chi^2))$ is the determinant of the Hessian matrix, which in turn is the inverse of the covariance matrix.

3. Results

Using the above analysis procedure, for each of the available photon energy bins, we fitted each data model to the angular distribution. This was carried out using the standard minimization package MINUIT. We then compared models with different numbers of Legendre polynomials by evaluating Eq. (4) for each data model.

To illustrate the procedure, we first choose one photon energy bin at $E_\gamma = 1.824$ GeV as an example. The posterior probabilities are shown in Fig. 1, where the order of the data model is shown on the horizontal axis. The maximum posterior is given by the data model containing four associated Legendre polynomials.

On the left side in Fig. 2 we show the fit of the fourth order data model to the CLAS differential cross section data⁶ for $E_\gamma = 1.824$ GeV. The procedure is repeated for each photon energy bin. The right side in Fig. 2 the order of data model which has the greatest probability at each photon energy is plotted. It can be seen that this generally increase from threshold into the resonance region, but that the maximum is mostly at the fourth order. The distributions of the polynomial coefficients for fourth order data models as a function of photon energy is shown in Fig. 3.

4 *P. T. P. HUTAURUK, et al.*

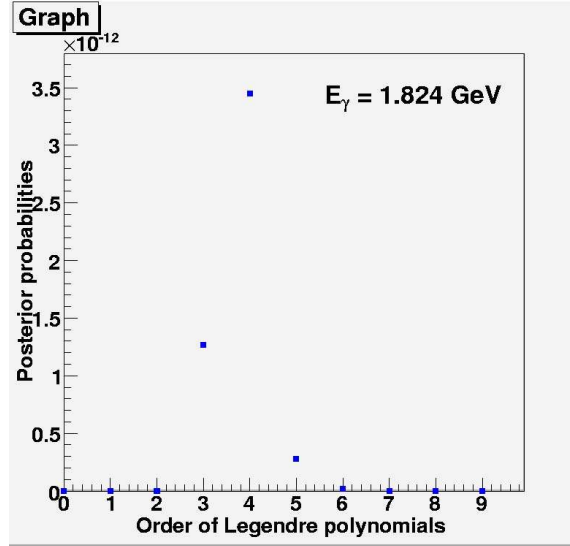


Fig. 1. The posterior probabilities for different orders of data model, for $E_\gamma = 1.824$ GeV.

4. Conclusion

We have analyzed the Legendre polynomial decomposition of differential cross section data. We generated data models with different numbers of associated Legendre polynomials. We then compared them by calculating posterior probabilities. From this analysis, we found that differential cross section data in this case requires at least four associated Legendre polynomials.

Acknowledgments

This work was supported by SUPA (Scottish Universities Physics Alliance) Fellowship.

References

References

1. C. G. Fasano and Frank Tabakin, Phys. Rev. C **46**, 6 (1992).
2. R. A. Adelseck and B. Saghai, Phys. Rev. C **42**, 108 (1990).
3. S. Capstick and W. Roberts, Phys. Rev. D **49**, 4570 (1994).
4. T. Mart and C. Benhold, Phys. Rev. C **61**, 012201R (2000).
5. T. Mart, Phys. Rev. C **62**, 038201 (2000).
6. R. Bradford *et al.* (CLAS collaboration), Phys. Rev. C **73**, 035202 (2006).
7. R. Bradford *et al.* (CLAS collaboration), Phys. Rev. C **75**, 035205 (2007).
8. K. -H. Glander *et al.* (SAPHIR collaboration), Eur. Phys. J. A **19**, 251-273 (2004).
9. M. Q. Tran *et al.* (SAPHIR collaboration), Phys. Lett. B **445**, 20 (1998).
10. K. -H. Glander *et al.* (SAPHIR collaboration), Nucl. Phys. A **754**, 294c-302c (2005).

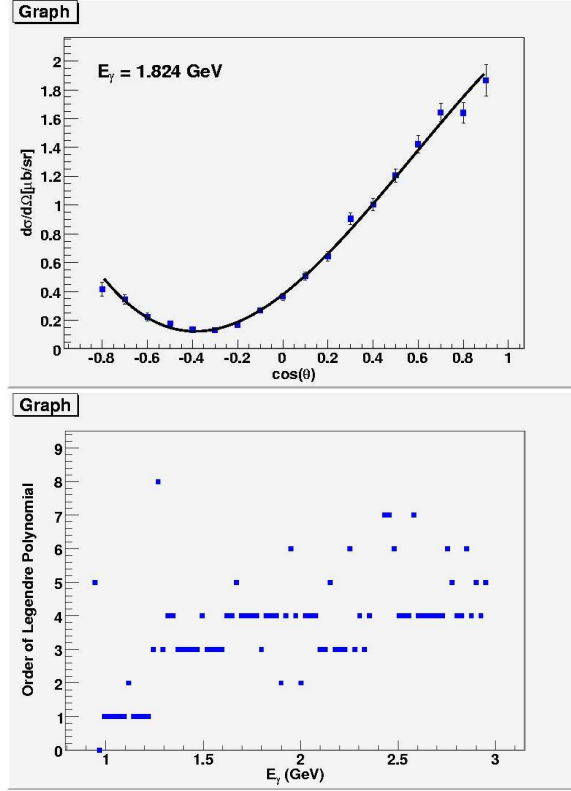


Fig. 2. Plot showing the fit of the fourth order data model to the CLAS cross section data for $E_\gamma = 1.824 \text{ GeV}$ (on the top). The order of associated Legendre polynomial for all photon energy (on the bottom).

11. M. Sumihama *et al.* (LEPS collaboration), Phys. Rev. C **73**, 035214 (2006).
12. R. G. T. Zegers *et al.* (LEPS collaboration), Phys. Rev. Lett. **91**, 9 (2003).
13. A. Lleres *et al.* (GRAAL collaboration), Eur. Phys. J. A **31**, 79-93 (2007).
14. D. G. Ireland, S. Janssen, J. Ryckebusch, Nucl. Phys. A **740**, 147-167 (2004).
15. Stijn Janssen *et al.*, Phys. Rev. C **65**, 015201 (2001).
16. S. Janssen *et al.*, Eur. Phys. J. A **11**, 105-111 (2001).
17. V. Shklyar *et al.*, Phys. Rev. C **71**, 055206 (2005) and references therein.
18. A. Usov and O. Scholten, Phys. Rev. C **72**, 025205 (2005).
19. G. Penner and U. Mosel, Phys. Rev. C **66**, 055211 (2002).
20. D. S. Sivia, Data Analysis: A Bayesian Tutorial. Clarendon Press. Oxford. (1996).

6 *P. T. P. HUTAURUK, et al.*

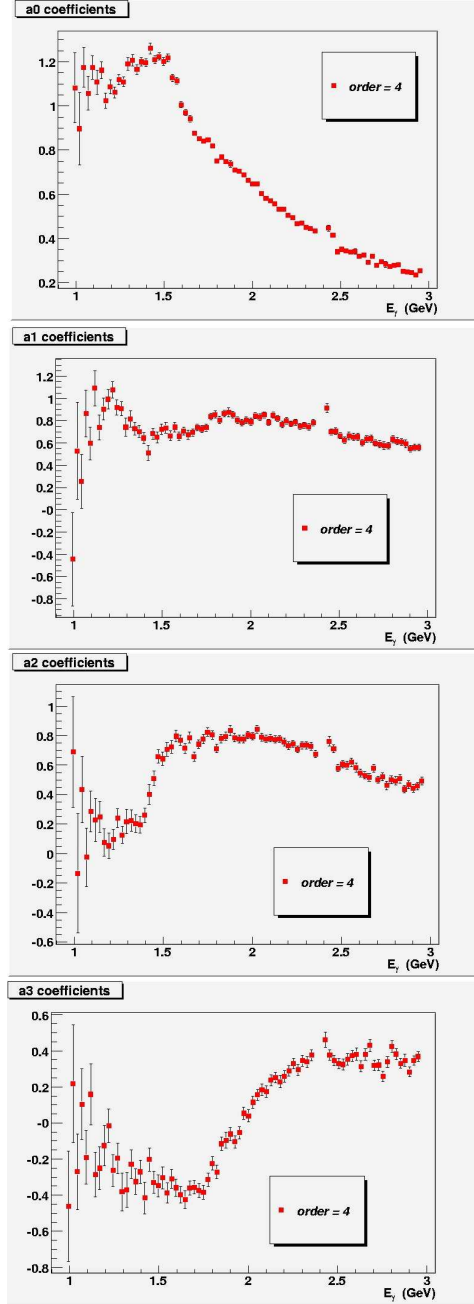


Fig. 3. Extracted associated Legendre polynomial coefficients for each photon energy.